LEVEL

(12)

FINAL REPORT

ARMA Spectral Estimation: An Efficient Closed Form Procedure

AD A106753

DTIC
SELECTED
NOV 6 1981

H

Prepared by:	Dr. James A. Cadzow
Academic Rank:	Professor
Department and University:	Department of Electrical Engineering Virginia Polytechnic Institute + State Univ. Blacksburg, VA. 24060
Prepared for:	Air Force Office of Scientific Research
Date:	August, 1981
Contract No.	AFOSR 80-0069

411062

DTIC FILE COPY

8 1 11 06 058

ARMA SPECTRAL ESTIMATION:

AN EFFICIENT CLOSED FORM PROCEDURE

by

JAMES A. CADZOW

ABSTRACT

In this report, an effective procedure for effecting on ARMA
spectral model of a time series is described. This procedure is
predicated on minimizing as set of "basic error" terms as generated
from an ARMA model that is hypothesized as characterizing the time
series under analysis. The spectral estimation performance achieved
in using this approach has been empirically found to be generally
superior to that obtained using such contemporary methods as maximum
entropy and the periodogram.

TABLE OF CONTENTS

Accession For

NTIS GRA&I	☑
DTIC TAB	☐
Unannounced	☐
Justification	

By

Distribution/

Availability Codes

Dist	Avail and/or Special
A	

I. INTRODUCTION

In this report, a description of the high performance ARMA spectral estimator, as in-part developed under the AFOSR contract 80-0069, shall be given. To begin, the main objective of the project was that of developing an effective method for estimating the a_k and b_k coefficients governing the ARMA model

$$x(n) + \sum_{k=1}^{p} a_k x(n - k) = \sum_{k-0}^{q} b_k \varepsilon(n - k) \qquad (1)$$

in which the excitation time series $\{\varepsilon(n)\}$ is taken to be white. These coefficients are to be selected so that this ARMA model is "most consistent" with a set of time series observations.

$$x(1), x(2), \cdots, x(N). \qquad (2)$$

which are available. The term "most consistent" is here being used in the sense that the hypothesized ARMA model is most compatable with the task of minimizing a set of "basic error terms". A brief description of this procedure shall be now given while a more detailed description is to be found in the appendix.

II. <u>DESCRIPTION OF ARMA MODELING METHOD</u>

The basis for the ARMA method is dependent on the so-called basic error terms. These terms are generated by first multiplying each side of relationship (1) by the delayed entity $x^*(n - M)$ to obtain

$$\epsilon(m,n) = x(n)x^*(n - m) + \sum_{k=1}^{p} a_k x(n - k)x^*(n - m) \qquad (3)$$

$$q + 1 \leq m < N$$

$$\max (m,p) \leq n \leq N$$

It can be shown that these basic error terms $\epsilon(m,n)$ are each zero mean when the underlying time series in an ARMA process of order less than or equal to (q,p) and the a_k coefficients in expression (3) correspond to the exact process' autoregressive coefficients.

With this in mind, a method for selecting the ARMA models a_k coefficients is suggested. Namely, they are chosen so as to cause the basic error terms (3) to be as close to their mean value of zero as possible. This objective may be realized by introducing the following quadratic functional

$$f(\underline{a}) = \underline{e}^\dagger W \underline{e} \qquad (4)$$

in which $\underline{e}$ is a column vector whose elements are appropriate arrangements of the ensemble of basic error terms (3), W is a nonegative matrix, $\underline{a}$ is the autoregressive coefficient vector with elements a_k, and, $\dagger$ denotes the operation of complex conjugate transposition. It is readily shown that the minimization of functional (4) will result in a set of consistent

2

linear equations for the optimum ARMA model autoregressive coefficients.

Once the a_k coefficients have been obtained, the ARMA model's moving average coefficients are estimated by first generating the so-called residual sequence $\{\varepsilon(n)\}$ according to

$$e(n) = x(n) + \sum_{k=1}^{p} a_k x(n - k) \qquad p + 1 \leq n \leq N \qquad (5)$$

The spectrum of this residual time series is theoretically given by $\left| \sum_{k=0}^{q} b_k e^{-jkw} \right|^2$. A technique for obtaining a moving average estimate of this residual time series is fully described in the appendix and entails the utilization of the smoothed periodogram. Once this smoothed periodogram has been generated, the required ARMA spectral estimate is achieved. Empirically derived results indicate that this report's procedure provides a superior spectral estimation performance than that achieved by such contemporary alternatives as the maximum entropy method, and the periodogram.

III. CONTRACT PUBLICATIONS

The following two refereed publications resulted from the sponsored AFOSR contract.

[1] " Autoregressive Moving Average Spectral Estimation: A Model
     Equation Error Procedure", IEEE Trans. on Geoscience and
     Remote Sensing, vol. GE-19, No. 1, Jan. 1981, pp 24 - 28.

[2] "Two Dimensional Spectral Estimation", IEEE Trans. on Acoustics,
     Speech and Signal Processing, vol. ASSP-29, No. 3, June 1981,
     pp 396 - 401.

Autoregressive Moving Average Spectral Estimation: A Model Equation Error Procedure

JAMES A. CADZOW, SENIOR MEMBER, IEEE

Abstract—A procedure is presented for generating an autoregressive moving average (ARMA) spectral model of a stationary time series based upon a finite set of time series' observations. The ARMA model's autoregressive coefficients are estimated by minimizing a quadratic function of a set of basic error terms. In examples treated to date, this method has demonstrated an exceptional ability in resolving closely spaced narrow band signals in a low signal-to-noise environment where other procedures such as the maximum entropy method often fail. Its effectiveness on other classes of time series also shows promise and a more general evaluation is presently being conducted. With this in mind, the new ARMA procedure promises to be an important spectral estimation tool.

I. INTRODUCTION

IN this paper, we shall be concerned with the task of estimating the statistical characteristics of a stationary random time series $\{x(n)\}$ from a finite set of observations. For many applications, knowledge of the time series' underlying autocorrelation sequence as formally defined by

$$r_x(n) = E\{x(n+k)x^*(k)\} \tag{1}$$

conveys all the information required. In this expression, E and * denote the expected value and complex conjugation operations, respectively. It is often advantageous to equivalently characterize stationary time series in the frequency domain where their intrinsic properties may be more discernible. This is particularly true for so-called narrow-band processes. The vehicle for this characterization is the associated power spec-

tral density as given by

$$S_x(\omega) = \sum_{n=-\infty}^{\infty} r_x(n) e^{-j\omega n} \tag{2}$$

which is recognized as being the Fourier transform of the autocorrelation sequence.

Upon examination of expressions (1) and (2), it is apparent that the required time series characterization will generally entail complete knowledge of the generally "infinite" length autocorrelation sequence. Here, we shall be concerned with extracting this information from the following "finite" set of time series observations

$$x(1), x(2), \cdots, x(N). \tag{3}$$

Unless some constraints are imposed upon the time series' basic nature, it is clear that there exists a fundamental incompatability in estimating the required statistical knowledge from this finite set of measurements. In accordance with well accepted practices, this dilemma is here resolved by postulating a finite parameter linear model to represent the random phenomenon. Specifically, the response of this model to "random" shock excitations will be conceived of as generating the time series under study.

In terms of parameter parsimony, the causal autoregressive moving average (ARMA) model of order (p, q) as specified by

$$x(n) + \sum_{k=1}^{p} a_k x(n-k) = \sum_{k=0}^{q} b_k \epsilon(n-k) \tag{4}$$

is generally the most effective linear model. In this representation, the unobserved excitation $\{\epsilon(n)\}$ is taken to be a white noise time series with zero mean and variance σ^2. It is important to appreciate the fact that the more specialized autoregressive (i.e., $b_k \equiv 0$ for $k \neq 0$) linear model will generally entail a much higher order choice so as to achieve a comparable statistical representation. Conceptually, the more efficient ARMA model is the logical model choice when the exact nature of the time series is unknown.

Due to the relatively difficult task of estimating the ARMA model's a_k and b_k parameters from the given observations (3), however, the preponderance of activity has been directed towards the specialized autoregressive (AR) model. This is a direct consequence of the simpler AR parameter estimation problem which results. In particular, the basically equivalent maximum entropy, linear predictive coding, and AR methods have been developed for efficiently obtaining the AR model parameters [1]. Nonetheless, the inherent superiority of an ARMA model representation is widely recognized and a number of procedures for estimating the ARMA model parameters have been advanced. These include the whitening filter approach which is iterative in nature, generally slow in convergence, and typically requires a relatively long data length N to be effective [2] and [3]. A more desirable closed-form approach which is based on the autocorrelation relationship governing ARMA processes has also been developed [4]-[6]. This approach often provides good estimates and does not require N to be excessively large. Unfortunately, its performance is not always as good as that provided by AR methods.

In this paper, a generalization of a recently developed closed-form ARMA method shall be presented [7] and [8]. This new procedure has been empirically found to provide significantly better spectral estimation performance than the above two ARMA approaches as well as the maximum entropy method. In what is to follow, a time domain approach for determining the ARMA model's AR a_k coefficients will be given. This in turn will be followed by a frequency domain procedure for estimating the effects of the moving average b_k coefficients on the overall spectral estimate. Use will be made of the well-known fact that the power spectral density corresponding to ARMA model (4) is specified by

$$S_x(\omega) = \frac{|b_0 + b_1 e^{-j\omega} + \cdots + b_q e^{-jq\omega}|^2}{|1 + a_1 e^{-j\omega} + \cdots + a_p e^{-jp\omega}|^2} \sigma^2$$

$$= \frac{|B(\omega)|^2}{|A(\omega)|^2} \sigma^2. \tag{5}$$

II. AUTOREGRESSIVE COEFFICIENT ESTIMATES

An effective method for estimating the ARMA model's AR coefficients from the set of given observations will be now presented. This first entails multiplying each side of expression (4) by the entity $x^*(n-m)$ to yield the "basic error terms"

$$e(m,n) = x(n) x^*(n-m) + \sum_{k=1}^{p} a_k x(n-k) x^*(n-m) \tag{6a}$$

$$= \sum_{k=0}^{q} b_k \epsilon(n-k) x^*(n-m), \quad \text{for } q+1 \leq m < N$$
$$\max (m,p) \leq n \leq N \tag{6b}$$

where the range on the m and n variables is dictated by the given time series range of $1 \leq n \leq N$ and the desire to have the random variables $\epsilon(n-k)$ and $x^*(n-m)$ be uncorrelated. If the time series is in fact an ARMA process of order less than or equal to (p, q) it follows that the basic error terms $e(m, n)$ are each zero mean random variables.[1] This is a consequence of the causality of model (4) and the whiteness of the excitation which results in the random variables $\epsilon(n-k)$ and $x^*(n-m)$ being uncorrelated.

With these thoughts in mind, relationship (6) provides an ideal vehicle for determining a set of AR coefficients which are consistent with the given time series observations. Specifically, the a_k coefficients will be selected so as to cause each of the basic error terms (6a) to be as close to their mean value of zero as possible. This goal is achieved by minimizing a squared error criterion of the form

$$f(a) = e^\dagger W e \tag{7}$$

in which e is a column vector whose elements are appropriate arrangements of the ensemble of basic error terms (6a). W is a

[1] As a side note, it may be shown that the basic error terms have identical variances given by

$$r_x(0) \sigma^2 \sum_{k=0}^{q} |b_k|^2.$$

nonnegative definite square matrix, and, † denotes the operation of complex conjugate transposition. This criterion is readily shown to be a function of the AR coefficients upon substitution of expression (6a) into criterion (7).

The weighting matrix W is to be selected based on statistical reasoning which must be tempered by an appreciation that little is generally known about the time series' statistics. To illustrate this point, let us consider the usually hypothesized condition that the excitation $\{\epsilon(n)\}$ is a Gaussian white process. In this case, if the matrix W is set equal to $E\{ee^\dagger\}^{-1}$, the minimization of criterion (7) will result in a maximum likelihood estimate of the AR coefficients. Unfortunately, the generation of this particular matrix requires knowledge of the time series' $\{x(n)\}$ second-order statistics. This statistical information, however, is precisely what we are seeking to estimate via the ARMA model (4).[2] This dilemma has arisen due to the unrealistic desire to seek a maximum likelihood ARMA model. More modest objectives must be sought if a tractable procedure is to be evolved.

A reasonable implementation of the above autoregressive coefficient selection philosophy then depends on a prudent choice for the weighting matrix. In this paper, we will be concerned exclusively with a choice that results in the following mean-squared error criterion:

$$f(a) = \sum_{m=q+1}^{N-1} w(m) \left| \sum_{n=1+\max(m,p)}^{N} e(m,n) \right|^2 \qquad (8)$$

although other choices are suggested in Section V. In this expression, the $w(m)$ are nonnegative parameters which are logically selected to be inversely proportional to the variance of the term which they multiply. A general expression for these variance entities, unfortunately, also reveals a dependency on the time series' unknown second-order statistics and the ARMA model coefficients. Empirical evidence suggests, however, that a choice of the weighting parameters as given by

$$w(m) = [N - m]^4 \qquad q + 1 \leqslant m \leqslant N - 1 \qquad (9)$$

results in satisfactory performance. An examination of other weighting parameter selections is currently being conducted.

Using standard calculus, it is readily shown that the set of AR coefficients which render criterion (8) a minimum must satisfy the following linear system of equations:

$$C\hat{a} = c \qquad (10)$$

where $\hat{a}$ is the $p \times 1$ AR coefficient vector estimate with elements a_k, C is a $p \times p$ nonnegative definite Hermitian matrix with elements

$$c_{lk} = \sum_{m=q+1}^{N-1} \sum_{n=t}^{N} \sum_{s=t}^{N} w(m) x(n-k) x(s-m)$$

$$\cdot x^*(n-m) x^*(s-i), \quad \text{for } i,k = 1,2,\cdots,p \qquad (11a)$$

[2] In the special case of an AR model, however, it is possible to obtain the maximum likelihood estimate.

where $t = 1 + \max(m,p)$ and c is a $p \times 1$ vector with elements

$$c_i = -c_{i0}, \quad \text{for } 1 \leqslant i \leqslant p. \qquad (11b)$$

Thus the optimum set of ARMA autoregressive coefficient estimates are obtained by solving the linear system of equation (10). It is possible to utilize the Hermitian structure of matrix C to yield an efficient solution procedure.

A. Forward and Backward Data Approach

One can more effectively use the observed data (3) to achieve an ARMA spectral estimate by using that data's backward version as specified by

$$\tilde{x}(1), \tilde{x}(2), \cdots, \tilde{x}(N) \qquad (12)$$

in which $\tilde{x}(n) = x^*(N + 1 - n)$. In particular, one formulates a mean-squared error criterion as given by

$$f(\hat{a}) = \sum_{m=q+1}^{N-1} w(m) \left\{ \left| \sum_{n=1+\max(m,p)}^{N} e(m,n) \right|^2 \right.$$

$$\left. + \left| \sum_{n=1+\max(m,p)}^{N} \tilde{e}(m,n) \right|^2 \right\} \qquad (13)$$

where $\tilde{e}(m, n)$ denotes the basic error term associated with the backward sequence (12). It is a simple matter to show that the AR coefficient vector which minimizes this forward-backward criterion must satisfy the linear system of equations

$$[C + \tilde{C}] \hat{a} = c + \tilde{c} \qquad (14)$$

where C and $\tilde{C}$ are each $p \times p$ nonnegative definite Hermitian matrices given by relationship (11a) with the forward (3) and backward (12) data used, respectively, and, the $p \times 1$ vectors c and $\tilde{c}$ are given similar interpretations.

It is important to note that the rank of matrix $[C + \tilde{C}]$ at least equals the rank of either of its constituent nonnegative definite matrices C or $\tilde{C}$. Thus, in using this forward and backward approach to estimate the AR coefficients, one is typically able to use a higher order model (i.e., larger value of p) than would be the case for the forward or backward models only. Moreover, it has been empirically found that the forward and backward approach usually results in better spectral estimates.

III. NUMERATOR DYNAMICS

A variety of procedures exist for determining the numerator dynamics of an ARMA time series once the AR coefficients have been estimated [4]–[9]. In this section, a procedure which has been found to produce improved results will be presented. It makes use of the characteristic equation (4) which can be interpreted as generating the auxiliary "residual" sequence $\{e(n)\}$ according to

$$e(n) = x(n) + \sum_{k=1}^{p} a_k x(n-k) \qquad (15a)$$

$$= \sum_{k=0}^{q} b_k \epsilon(n-k). \qquad (15b)$$

One may straightforwardly show that the spectrum of the moving average residual time series $S_e(\omega)$ is given by $|B(\omega)|^2 \sigma^2$.

This observation in conjunction with relationship (5) provides the vehicle for obtaining the underlying time series spectral estimate, that is

$$S_x(\omega) = S_e(\omega)/|A(\omega)|^2. \qquad (16)$$

With this in mind, a method will be now given for obtaining a qth-order moving average spectral estimate of the residual sequence. This estimate will be based on the specific residual elements

$$e(p+1), e(p+2), \cdots, e(N) \qquad (17)$$

which are calculated using relationship (15a) and the given time series observations (3). The AR coefficient used in (15a) will correspond to the a_k estimate obtained upon solving expression (14).

The approach to be now presented in an adaption of the well-known method of Welch for obtaining smoothed periodogram estimates [10]. In essence, one first segments the calculated residual elements (17) into L segments each of length $q+1$ as specified by

$$e_k(n) = \alpha(n)\, e(n+1+p+kd), \quad 0 \leq n \leq q$$
$$0 \leq k \leq L-1 \qquad (18)$$

where $\alpha(n)$ is a data window and "d" a positive integer which specifies the time shift between adjacent segments. These individual segments will overlap for a shift selection of $d \leq q$. Furthermore, to include only observed data, the relevant parameters must be selected so that $p+q+1+(L-1)\,d \leq N$. Finally, the periodogram of each of the L segments is taken and these are averaged to obtain the desired smoothed qth-order periodogram, that is

$$\hat{S}_e(\omega) = \frac{1}{L} \sum_{k=0}^{L-1} \left\{ \frac{1}{q+1} \left| \sum_{n=0}^{q} \alpha(n)\, e(n+p+kd)^{-j\omega n} \right|^2 \right\}. \qquad (19)$$

The data window is to be normalized according to $\sum \alpha^2(n) = 1$. In using this segmentation, the variance of the estimate $S_e(\omega)$ is reduced. The price paid for this reduction, however, is a loss in frequency resolution, an increased bias of the estimate, and, a possible deterioration in power level estimates. It must be noted, however, that the overall resolution capability of this ARMA procedure is predominately influenced by the AR coefficient selection. If one is basically interested in resolution performance, an examination of the ARMA model's pole locations then need only be considered.

IV. NUMERICAL EXAMPLE

In order to compare the effectiveness of the new ARMA spectral estimator with the maximum entropy method, the classical problem of resolving two closely spaced (in frequency) sinusoids in white noise will be considered. Specifically, the time series under study is specified by

$$x(n) = \sqrt{20} \cos(0.4\pi n) + \sqrt{2} \cos(0.426\pi n) + w(n) \qquad (20)$$

where $\{w(n)\}$ is a white Gaussian noise process of zero mean and unity variance. The sinusoids of normalized frequencies
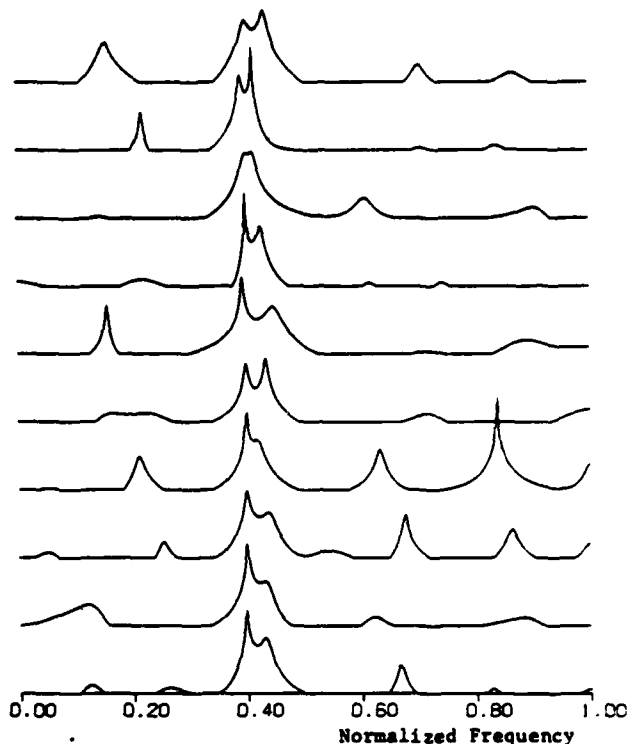

Fig. 1. Fifteenth-order ARMA spectral estimate of two sinusoids at normalized frequencies $f_1 = 0.4$ (10 dB) and $f_2 = 0.426$ (0 dB) based upon individual sequences of length 64.
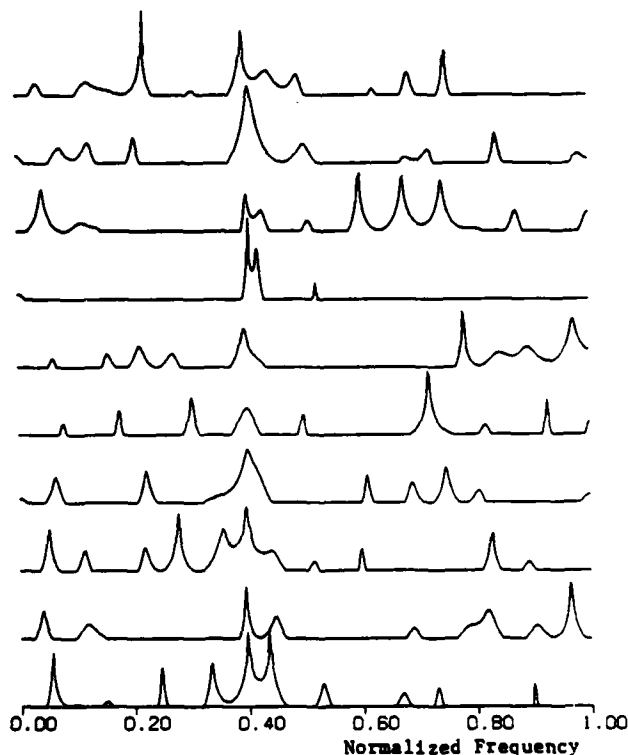

Fig. 2. Thirtieth-order maximum entropy AR spectral estimate of two sinusoids at normalized frequencies $f_1 = 0.4$ (10 dB) and $f_2 = 0.426$ (0 dB) based upon individual sequences of length 64.

0.4 and 0.426 are readily calculated to have a signal-to-noise ratio (SNR) of 10 dB and 0 dB, respectively. A sequence of
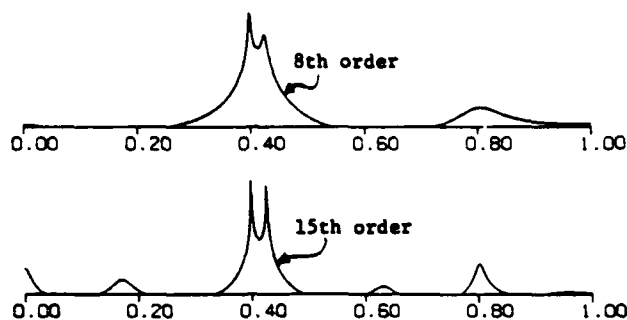
Fig. 3. ARMA spectral estimates of two sinusoids at normalized frequencies $f_1 = 0.4$ (10 dB) and $f_2 = 0.426$ (0 dB) based upon all 640 observations.
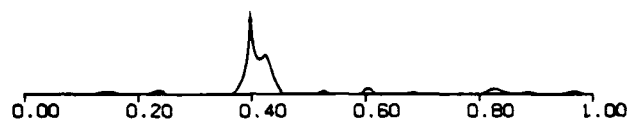

Fig. 4. Thirtieth-order maximum entropy AR spectral estimate of two sinusoids at normalized frequencies $f_1 = 0.4$ (10 dB) and $f_2 = 0.426$ (0 dB) based upon all 640 observations.

length 640 defined over $0 \leqslant n \leqslant 639$ was next generated using this relationship. Furthermore, in order to provide a statistical basis for our comparison, this 640 length sequence was then decomposed into ten disjoint sequences each of length 64 defined on $0 \leqslant n \leqslant 63$, $64 \leqslant n \leqslant 127, \cdots, 576 \leqslant n \leqslant 639$. An ensemble consisting of ten subsequences each of length 64 has thereby been generated with each subsequence having a different noise sample and a different initial phase between the two sinusoids. This latter condition is useful in revealing any potential sensitivity to initial phase that the new ARMA spectral estimation method may possess.

The spectral estimates which resulted when the new ARMA spectral estimator (forward data only) and the maximum entropy (covariance) method were applied to these ten random time series samples are displayed in Figs. 1 and 2, respectively. The ordinates were scaled from 0 to 50 dB for each of the ten individual plots with each AR maximum entropy estimate being of order thirty and each ARMA estimate being of order fifteen (with $d = 16$). The spectral estimates are plotted one above the other (lower one corresponds to $0 \leqslant n \leqslant 63$) so as to better reveal the consistency of the estimates and to conserve space. It is clear that the maximum entropy method is able to resolve the sinusoids in only five of the ten sample cases. On the other hand, the new ARMA method successfully resolves the sinusoids in each case and provides excellent frequency estimates of $\hat{f}_1 = 0.3980$ and $\hat{f}_2 = 0.4295$ obtained by averaging over the ten cases with associated sampled standard deviations of $\sigma_{\hat{f}_1} = 0.0027$ and $\sigma_{\hat{f}_2} = 0.0107$.

To illustrate the new ARMA method's effectiveness on long data length sequences, an eighth- and fifteenth-order ARMA model were next obtained using the entire sequence of length 640 as given above. The spectral estimates which resulted are displayed in Fig. 3 where a sharpness in frequency resolution is

apparent. In addition, a thirtieth-order AR spectral estimate which arises from this 640-length sequence is shown in Fig. 4. Clearly, the new ARMA method has outperformed the AR maximum entropy method in both the short and long data length examples here considered.

V. CONCLUSIONS

A generalization of a recently developed ARMA spectral estimation method has been presented. This has included the introduction of an error weighting matrix, the concept of using forward and backward data, and, the utilization of Welch's method for obtaining estimates of the spectrum's numerator dynamics. Empirical evidence suggests that this new procedure has an improved spectral resolution performance when compared to popular contemporary methods. Its full potential will be realized, however, only after a number of relevant issues are resolved. These include the task of determining good choices for the weighting matrix W, obtaining a data-dependent procedure for estimating the ARMA model order, and, investigating other numerator dynamics methods.

In addition to the specific weight matrix selection considered in this paper, the following two choices of criterion

$$\sum_{m=q+1}^{N-1} \sum_{m=m+1}^{N} |e(m,n)|^2$$

and

$$\sum_{n=q+2}^{N-1} w(n) \left| \sum_{m=q+1}^{N-1} e(m,n) \right|^2$$

have given preliminary evidence of providing satisfactory spectral estimation performance. Further investigation of this most critical weighting matrix choice is now being conducted and will be subsequently reported upon.

REFERENCES

[1] D. G. Childers, *Modern Spectral Analysis*. New York: IEEE Press, 1978.
[2] P. R. Gutowski, E. A. Robinson, and S. Treitel, "Spectral estimation, fact or fiction," *IEEE Trans. Geosci. Electron.*, vol. GE-16, pp. 80–84, Apr. 1978.
[3] K. Steiglitz, "On the simultaneous estimation of poles and zeros in speech analysis," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-25, pp. 229–234, June 1977.
[4] G. Box and G. Jenkins, *Time Series Analysis: Forecasting and Control*, revised edi. San Francisco, CA: Holden-Day, 1976.
[5] M. Kaveh, "High resolution spectral estimation for noisy signals," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-27, pp. 286–287, June 1979.
[6] J. F. Kinkel, J. Perl, L. Scharf, and A. Stubberud, "A note on covariance–Invariant digital and autoregressive moving average spectrum analysis," *IEEE Trans. Acoust., Speech, and Signal Processing*, vol. ASSP-27, pp. 200–202, Apr. 1979.
[7] J. A. Cadzow, "ARMA spectral estimation: An efficient closed form procedure," in *Proc. Rome Air Development Center Workshop Spectral Estimation* (Rome, NY, Oct. 3–5, 1979), pp. 81–97.
[8] ——, "High performance spectral estimation: A new ARMA method," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-28, pp. 524–529, Oct. 1980.
[9] S. M. Kay, "A new spectral estimator," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-28, pp. 585–588, Oct. 1980.
[10] S. A. Tretter, *Discrete-Time Signal Processing*. New York: Wiley, 1976.

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
AFOSR-TR-81-0727	AD-A106753	

4. TITLE (and Subtitle)

ARMA SPECTRAL ESTIMATION: AN EFFICIENT CLOSED FORM PROCEDURE.

5. TYPE OF REPORT & PERIOD COVERED

FINAL rept

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)

Dr. James A. Cadzow

8. CONTRACT OR GRANT NUMBER(s)

AFOSR-80-0069

9. PERFORMING ORGANIZATION NAME AND ADDRESS

Department of Electrical Engineering
Virginia Polytechnic Institute
Blacksburg, VA. 24060

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

61102F
2305-D9

11. CONTROLLING OFFICE NAME AND ADDRESS

AFOSR of Scientific Research
Building 410
Bolling AFB, Washington, D.C. 20332

12. REPORT DATE

August 1981

13. NUMBER OF PAGES

11

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)

15. SECURITY CLASS. (of this report)

Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release;
distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A procedure was devised for generating an autoregressive moving average (ARMA) spectral model of a stationary time series based upon a finite set of time series' observations. The ARMA model's autoregressive coeficients are estimated by minimizing a quadratic function of a set of basic error terms. In examples treated to date, this method has demonstrated an exceptional ability in resolving closely spaced narrow band signals in low signal-to-noise environment where other procedures such as the maximum entropy method often fail. -OVER -

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE

1 JAN 73

Unclassified

Its effectiveness on other classes of time series also shows promise and a more general evaluation is presently being conducted. With this in mind, the new ARMA procedure promises to be an important spectral estimation tool.

END

DATE
FILMED

11—81

DTIC